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A group of agents must defend their individual income from an external threat by pooling their efforts against it. The winner of this confrontation is determined by a contest success function where members' efforts may display different degrees of complementarity. Individual effort is costly and follows a convex isoelastic function. We investigate how the success of the group in the conflict and its members' utilities vary with the degree of within-group inequality. We show that there is a natural relationship between the group's probability of victory and the Atkinson index of inequality. If members' efforts are complementary or the cost function convex enough, more egalitarianism within the group increases the likelihood of victory against the external threat. The opposite holds when members' efforts are substitutes and the cost linear enough. Finally, we obtain conditions under which richer members of the group are willing to make transfers to poorer members in order to enhance their final payoff.

Keywords: Conflict, Inequality, Atkinson index, Redistribution.
JEL codes: D31, D63, D72, D74.

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1 Introduction

Societies and communities often have to defend from or compete with hostile out-groups. Cities and villages often suffered raids from barbarians, pirates or bandits. Ordinary citizens need to protect themselves from the appropriation efforts by criminal networks. States and empires often clash over the control of natural resources or have to resist the attack of rival nations. Suppose that these communities, while remaining in confrontation with an out-group, have solved any possible conflict of interests within themselves. They accept the current distribution of income or hold binding agreements on how to share the value of the object they are fighting for against the out-group. We then ask the following question: Are more egalitarian societies more or less likely to prevail in such confrontations?

Apart from its intrinsic interest, this question is important because the interplay between chances of success and within-group inequality opens the door to income redistribution. If, for instance, a more egalitarian distribution of income within the community enhances its prospects of victory, members of that society may voluntarily transfer part of their income to poorer members. In short, the presence of external conflicts may provide a rationale for the redistribution of income we observe in societies.

In this paper we show that the answers to these two questions, whether egalitarianism enhances the chances of victory of society and whether a society may want to engage in income redistribution as a result, depend on the technologies of conflict. If the efforts of the members are substitutes, more inequality is better because inequality increases the incentives to contribute of richer members, who are the ones who have most to gain from victory. If, on the contrary, efforts are complements, more egalitarian societies fare better in the confrontation. This is because all members must contribute for society to be successful in the conflict, implying that members with the lowest incentives to contribute are key. These members are the poorer members since they are the ones with the lowest stake in the fight. So the richer they are, that is, the more egalitarian is the distribution of income within that society, the more they contribute to defeat the out-group. The cost of conflict contributions plays also a crucial role. If the marginal cost of efforts increases rapidly, this will deter richer members from contributing substantially. In that case, more egalitarianism makes the group more effective in the confrontation.

Conflict thus generates two types of redistribution. First, conflict shapes the income distribution within society because its members contribute to the success of the group. The resulting distribution of income could be more or less egalitarian than the initial one. We show that as efforts become more complementary the distribution of contributions becomes more egalitarian so conflict is in effect implementing a regressive tax scheme. As a result, the final distribution of income is more unequal than the initial one. We say then that conflict is *pro-rich*. The opposite holds when efforts are substitutes. Conflict is *pro-poor* because richer members contribute a bigger share of their income than poorer members. Second, members of the community may be willing to engage in income redistribution as a result of the presence of the hostile out-group. The incentives to redistribute vary in their direction, from the rich to the poor or viceversa, depending on the technologies of conflict. When efforts are complementary enough or their cost is convex enough we show that richer members are willing to transfer *voluntarily* part of their income to poorer members. The opposite holds when efforts are substitutes or the cost is linear enough. This is because even though success of the group increases with inequality in that case, poorer members have less to gain from victory. They have little incentives to transfer their income to richer members in order to induce them to fight harder.

The relationship between egalitarianism and collective action has been subject to analysis for long now. Olson (1965) argued informally that more inequality favors collective action, public good provision for instance, since it maximizes the incentives of richer members to engage in it. Hirshleifer (1983) argued that this result rests critically on the assumption that the amount of public good provided depends on the sum of contributions. When contributions are perfect complements, i.e., the weakest-link technology of provision, inequality hinders public good porvision. Using examples, Cornes (1993) and Cornes and Sandler (1996) corroborated that Olson's intuition does not hold in general. In the closest contribution to ours, Ray et al. (2007) characterize the relationship between the surplus generated by a joint project, inequality in the shares of the resulting output and the technology of production. These authors show, as we do, that egalitarianism can be welfare enhancing if contributions are complementary enough. We focus on the specific case of conflict against an out-group and consider the incentives to redistribute income that communities may have as a result. In the context of public goods, Vicary (1990) and Cornes (1993) explored this issue but only for the weakest-link technology. Both show that under this technology the "distribution neutrality" result by Bergstrom et al. (1986) no longer holds.

In the literature on conflict, to the best of our knowledge, only Esteban and Ray´s (2011) model of ethnic conflict has analyzed the role of within-group inequality. These authors model a situation where members can contribute with their time or use money to increase the activism of other members. Money and time are thus substitutes. They show that more within-group inequality makes groups more violent because the opportunity cost of time for poorer member decreases, so richer members find it easier to buy higher levels of activism from them. Our model also belongs to the literature that has explored the role of conflict in producing income redistribution. Hirshleifer (1991) pointed out that income redistribution is one of society's responses to the threat of internal conflict. Individuals or social groups resort to conflict if by doing so they can improve their position relative to the current distribution of income. Within-society income redistribution thus becomes a way of avoiding internal conflict. Bevia and Corchon (2010) argued that a similar mechanism can help societies to avoid conflict against external agents. If a rich society transfers some of its income to an external group, that out-group becomes less interested in initiating a costly confrontation. In our case, we study how redistribution within a society can help to improve the society's chances of victory in an external confrontation.

The rest of the paper is organised as follows. In the next section we present the basic elements of the model. In Section 3, we characterize its equilibrium and perform comparative statics. Section 4 explores the implicit redistribution that conflict generates and the incentives of members of the group to engage in income transfers. Section 5 concludes.

2 The model

2.1 Conflict

Let us consider a group formed $n > 1$ members who differ in income. A member i owns a share $\alpha_i > 0$ of the total income of the group (net of subsistence level) denoted by Y such that $\sum_{i=1}^{n} \alpha_i = 1$. Let us index members increasingly by income so $\alpha_i \leq \alpha_{i+1}$ for $i = 1, ..., n-1$.

This group is subject to the threat of an external entity that we model as a unitary agent. We will simply refer to it as the threat. Both the group and the threat are in confrontation. If the group prevails its members are able to retain their individual income $y_i = \alpha_i Y$. If the threat wins the conflict, it appropriates the entire income of the group and its members get nothing. Alternatively we could interpret the setting as a situation where the group and the threat are competing for a prize of value Y (a territory, a monopoly rent, a natural resource) so the vector $\boldsymbol{\alpha} = (\alpha_1, ..., \alpha_n)$ represents a binding agreement among group members on how to divide that prize in case of victory.

Both the group and the Threat can invest resources in order to prevail in this confrontation. The outcome of the conflict depends on the efforts spent by each of the two sides. Denote by $\mathbf{x} = (x_1, ..., x_n)$ the vector of conflict efforts made by the members of the group and denote by x_o the effort made by the threat. We assume that the group's winning probability is

$$p(\mathbf{x}) = \frac{h(\mathbf{x},n)}{x_o + h(\mathbf{x},n)}, \tag{1}$$

where the function

$$h(\mathbf{x},n) = n[\sum_{i=1}^{n} \frac{1}{n} {x_i}^{1-\sigma}]^{\frac{1}{1-\sigma}}, \tag{2}$$

is called the *impact function* of the group. The parameter $\sigma \geq 0$ represents the degree of complementarity between members' efforts. In the context of public good provision, similar functions have been used by Cornes (1993) and Ray et al. (2007). On the other hand, the group Contest Success Function (CSF) in (1) has been axiomatized by Münster (2009). It encompasses as particular cases the Tullock CSF (Tullock, 1967) when $\sigma = 0$ and the weakest-link technology (Hirshleifer, 1983) when $\sigma \to \infty$.

In particular, note that impact function (2) satisfies two important properties:

- **Constant returns to scale:** For all $k > 0$, $h(k\mathbf{x}, n) = kh(\mathbf{x}, n)$.
- **No group-size bias** (Kolmar and Rommeswinkel, 2011): For any natural number k, it holds that $h(\frac{\mathbf{x}}{k}, kn) = h(\mathbf{x},n)$.

Constant returns to scale is an appealing property in this context because it implies that the relative success of a contender does not depend on the unit of measurement of effort. Note that this property also implies that the CSF in (1) is homogeneous of degree zero (Münster, 2009). On the other hand, the No group-size bias property implies that the impact of two groups who have exerted the same total effort should be the same regardless of their size. Many CSFs implicitly build in some group-size bias because this property is closely related to the degree of complementarity of efforts. Consider for instance the seemingly more natural impact function.

$$g(\mathbf{x},n) = [\sum_{i=1}^{n} {x_i}^{1-\sigma}]^{\frac{1}{1-\sigma}}.$$

It is straightforward to show that this impact function presents *positive* group-size bias, i.e. $h(\frac{\mathbf{x}}{k}, kn) > h(\mathbf{x},n)$, if and only if $\sigma < 1$, and a *negative* group-size bias, i.e. $h(\frac{\mathbf{x}}{k}, kn) < h(\mathbf{x},n)$, if and only if $\sigma > 1$. By assuming away any group-size bias, we avoid any confounding effect from group size on our result and focus only on the effect of internal inequality.

We assume that the cost of effort is iso-elastic and of the form

$$c(x_i) = \frac{1}{1+\phi} x_i^{1+\phi},$$

where $\phi \geq 0$. Similarly for the threat. This functional form was first considered by Esteban and Ray (1999) and its properties studied in relation to the group-size paradox in Esteban and Ray (2001).

The payoff function for a group member boils down to

$$u_i = py_i - c(x_i) = \frac{h(\mathbf{x},n)}{x_o + h(\mathbf{x},n)} \alpha_i Y - \frac{1}{1+\phi} x_i^{1+\phi}, \tag{3}$$

whereas for the threat it is just

$$u_o = (1-p)Y - c(x_o) = \frac{x_o}{x_o + h(\mathbf{x},n)} Y - \frac{1}{1+\phi} x_o^{1+\phi}. \tag{4}$$

2.2 Inequality

Consider that the income distribution in a society is given by the vector $\mathbf{y} = (y_1, ..., y_n)$. The measure of income inequality we will consider here was proposed by Atkinson (1970) and it is defined as

$$A_\varepsilon(\mathbf{y}) = 1 - \frac{y_\varepsilon}{\overline{y}},$$

where $\overline{y}$ is society's average income and y_ε is the *Equally Distributed Equivalent Income* (EDEI) which is given by

$$y_\varepsilon = [\frac{1}{n} \sum_{i=1}^{n} y_i^{1-\varepsilon}]^{\frac{1}{1-\varepsilon}} = Y \cdot [\frac{1}{n} \sum_{i=1}^{n} {\alpha_i}^{1-\varepsilon}]^{\frac{1}{1-\varepsilon}}. \tag{5}$$

The parameter ε measures society's attitude towards inequality. In our set up, the Atkinson index boils down to

$$A_\varepsilon(\mathbf{y}) = \begin{cases} 1 - n[\frac{1}{n} \sum_{i=1}^{n} {\alpha_i}^{1-\varepsilon}]^{\frac{1}{1-\varepsilon}} & \text{for } \varepsilon \neq 1 \\ 1 - n \prod_{i=1}^{n} \alpha_i^{\frac{1}{n}} & \text{for } \varepsilon = 1 \end{cases}. \tag{6}$$

The parameter ε embeds a normative judgment over income inequality.[1] Observe that $A_\varepsilon(0) = 0$ and that $\lim_{\varepsilon\to\infty} A_\varepsilon(\mathbf{y}) = 1 - n\alpha_1$, so the inequality index depends only on the income of the worst-off member in the group. In the original formulation of the Atkinson index, it is always assumed that $\varepsilon \geq 0$ so incomes are socially evaluated according to a concave function, implying $y_\varepsilon < \overline{y}$. Under that assumption, the index is consistent with the following principle:

- **Principle of transfers (Pigou-Dalton principle):** Take two vectors $\mathbf{y}$ and $\mathbf{y'}$, where $\mathbf{y'}$ is obtained by adding $\Delta > 0$ to y_i and substracting it from y_j for $j > i$ and such that $y_j - \Delta > y_i + \Delta$. Then $A_\varepsilon(\mathbf{y}) > A_\varepsilon(\mathbf{y'})$.

This principle states that when $\varepsilon \geq 0$ a rank-preserving income transfer from a richer individual to a poorer individual cannot increase inequality. However, the functional form of the Atkinson index does not preclude that society may have a *preference* for inequality, i.e. $\varepsilon < 0$. The literature on inequality measurement never considers this case since this literature implicitly assumes that inequality is not socially desirable. When $\varepsilon < 0$ it turns out that $y_e > \overline{y}$, and the index becomes non-positive, with higher absolute values corresponding to higher levels of inequality. Observe for instance that $\lim_{\varepsilon\to-\infty} A_\varepsilon(\mathbf{y}) = 1 - n\alpha_n$, so the index is negative unless the distribution of income is perfectly equal, i.e. $\alpha_i = \frac{1}{n}$ for all members. In this case thus, the index satisfies the Reversed principle of transfers, that is, given two distributions $\mathbf{y}$ and $\mathbf{y'}$ defined as above $A_\varepsilon(\mathbf{y}) < A_\varepsilon(\mathbf{y'})$.

It is easy to show that for any value of ε, either positive or negative, the Atkinson index satisfies two particularly relevant properties (Lambert, 2001).

- **Scale invariance:** For all $k > 0$, $A_\varepsilon(k\mathbf{y}) = A_\varepsilon(\mathbf{y})$.

- **Principle of population:** For any natural number k denote by $\mathbf{y}^k$ the vector containing k times each and all of the elements in $\mathbf{y}$. Then $A_\varepsilon(\mathbf{y}^k) = A_\varepsilon(\mathbf{y})$.

[1] Atkinson (1970) proposes an equivalence between inequality aversion and risk aversion based on the idea that behind the veil of ignorance more risk-averse individuals would prefer more egalitarian distributions of income. Under that interpretation, the EDEI is the level of income that if equally distributed would give individuals a level of equality equal to the expected utility they would enjoy under the original distribution behind the veil of ignorance .

Scale invariance, i.e. homogeneity of degree zero, is an appealing property because it implies that inequality does not depend on the unit of measurement of income. On the other hand, the Principle of population implies that the Atkinson index remains invariant under replications of the population (and its incomes). These properties will be helpful later when characterizing the equilibrium of the conflict game.

3 The equilibrium

3.1 Existence

Let us now characterize the equilibrium of this game. To do so we exploit the properties of the Atkinson index we have just outlined.

A member i of the group seeks to maximize (3) taking as given the effort of other members and the effort made by the threat. Her optimal decision is characterized by the following expression

$$\frac{\partial u_i}{\partial x_i} = \frac{p(1-p)}{h(\mathbf{x},n)} \frac{\partial h(\mathbf{x},n)}{\partial x_i} \alpha_i Y - x_i^{\phi} = 0 \qquad i = 1, ..., n. \tag{7}$$

From this it is possible to write the relation between the optimal efforts of any two members

$$\frac{x_i}{x_j} = (\frac{\alpha_i}{\alpha_j})^{\frac{1}{\phi+\sigma}}. \tag{8}$$

This expression give us a first indication of how the impact function and the cost function affect the distribution of efforts across members. Member's efforts become more similar the more complementary efforts are and the more convex their cost, i.e. the higher $\phi + \sigma$. Actually, expression (7) implies that whenever $\phi + \sigma > 0$ it cannot be a best response for a member to exert no effort if another member is exerting a positive effort. Hence, in any equilibrium, either all members or no member are contributing. There are, however, two exceptions to this result.

Case 1: Tullock contest (Tullock, 1967) Consider the case where members' efforts are perfect substitutes and the cost is linear, i.e. $\phi+\sigma = 0$. In that case, observe that the first order condition (7) becomes

$$\frac{\partial u_i}{\partial x_i} = \frac{p(1-p)}{\sum_{i=1}^{n} x_i} \alpha_i Y - 1 \qquad i = 1, ..., n,$$

implying that only the member with the highest income, that is, member n, exerts positive effort. This is a well-known result in the literature on contests (Baik, 1993).

Case 2: Weakest-link technology (Hirshleifer, 1983) When the impact function displays perfect complementarity between members' efforts, i.e. $\sigma \to \infty$, it boils down to

$$h(\mathbf{x},n) = n \cdot \min\{x_1, ..., x_n\}.$$

In that case, it is clear that in any equilibrium all members will exert the same level of effort. Define $\overline{x}_1$ as the effort choice of the poorest member that satisfies

$$\frac{\partial u_i}{\partial x_i} = \frac{x_o}{(n\overline{x}_1 + x_o)^2}\alpha_1 Y - 1 = 0,$$

that is, $\overline{x}_1$ is member 1 optimal choice under the assumption that all other members also exert $\overline{x}_1$. Then, it is quite straightforward to see that there exists a continuum of equilibria in which all members contribute the same amount of effort, ranging from 0 to $\overline{x}_1$. Members of the group face thus a coordination problem.

For the sake of exposition we assume that $\phi + \sigma > 0$ and focus on fully interior equilibria.

On the other hand, the optimal effort choice for the threat is given by the following FOC

$$\frac{\partial u_i}{\partial x_o} = \frac{p(1-p)}{x_o}Y - x_o^{\phi} = 0. \tag{9}$$

Combining (7) and (9) it is possible to express the relationship between the optimal effort decisions of the group members and the threat in a compact way

$$\frac{c(x_i)}{c(x_o)} = (\frac{x_i}{x_o})^{1+\phi} = \frac{\alpha_i x_i}{h(\mathbf{x},n)}\frac{\partial h(\mathbf{x},n)}{\partial x_i} = \frac{\alpha_i^{\frac{1+\phi}{\phi+\sigma}}}{\sum_{j=1}^{n} \alpha_j^{\frac{1-\sigma}{\phi+\sigma}}} \tag{10}$$

We are finally in the position to state our first result

Proposition 1 *When $\phi + \sigma > 0$, there exists a unique interior equilibrium effort profile characterized by*

$$\begin{aligned} x_o^{1+\phi} &= p(1-p)Y, \\ x_i^{1+\phi} &= p(1-p)t_i y_i \qquad i = 1, ..., n, \end{aligned}$$

where

$$t_i = \frac{\alpha_i^{\frac{1-\sigma}{\phi+\sigma}}}{\sum_{j=1}^{n} \alpha_j^{\frac{1-\sigma}{\phi+\sigma}}} \tag{11}$$

Proof. First, we need to show that (7)-(9) characterize local maxima of the maximization problem faced by member i and the external threat respectively. For the threat, the SOC is satisfied if

$$\frac{\partial^2 u_i}{\partial^2 x_o} = -2\frac{p(1-p)^2}{x_o^2}Y - \phi x_o^{\phi-1} < 0,$$

which is always the case. For the problem of a member i, the SOC is satisfied if

$$\frac{\partial^2 u_i}{\partial^2 x_i} = \frac{p(1-p)}{h(\mathbf{x},n)}\alpha_i Y[\frac{\partial^2 h(\mathbf{x},n)}{\partial^2 x_i} - \frac{2}{H}(\frac{\partial h(\mathbf{x},n)}{\partial x_i})^2] - \phi x_i^{\phi-1} < 0 \qquad i = 1, ..., n, \tag{12}$$

where in order to ease notation $H = h(\mathbf{x},n) + x_o$. Simple calculus shows that

$$\frac{\partial^2 h(\mathbf{x},n)}{\partial^2 x_i} = \sigma x_i^{-\sigma-1}(\frac{h(\mathbf{x},n)}{n})^{\sigma}[\frac{x_i^{1-\sigma}}{\sum_{i=1}^n x_i{}^{1-\sigma}} - 1] \leq 0.$$

which implies that (7) holds strictly whenever $\sigma + \phi > 0$ and there is at most one value of x_i that solves this equation for any given x_{-i} and x_o. ■

This Proposition demonstrates: members' effort contributions are non-decreasing in their income, i.e. $t_i y_i > t_j y_j$ for all $i > j$. Richer members have a higher marginal benefit from contributing to the success of the group and hence exert more effort. On the other hand, observe that members contribute a share t_i of their income. The presence of an external conflict is thus taxing members implicitly. This implicit tax t_i is not raised by any authority. It is paid voluntarily by members in order to defend their incomes. This tax rate does not necessarily increase with the income of the member, so it is possible that poorer members make lower contributions but they contribute a higher share of their income. Below we exploit this result and analyze whether the implicit redistribution that conflict generates is *pro-rich* or *pro-poor*.

From here, and using simple algebra, it is possible to state one of our main results.

Proposition 2 *The group's equilibrium winning probability is*

$$p^* = \frac{[1 - A_{\widehat{\varepsilon}}(\mathbf{y})]^{\frac{1}{1+\phi}}}{n^{\frac{1-\phi}{1+\phi}} + [1 - A_{\widehat{\varepsilon}}(\mathbf{y})]^{\frac{1}{1+\phi}}}, \tag{13}$$

where $\widehat{\varepsilon} = 1 - \frac{1-\sigma}{\phi+\sigma}$.

Proof. Combining the expressions (8) and (10), it is possible to write the equilibrium winning probability of the group as

$$p^* = \frac{[[\sum_{j=1}^{n} \alpha_j^{\frac{1-\sigma}{\phi+\sigma}}]^{\frac{\phi+\sigma}{1-\sigma}}]^{\frac{1}{1+\phi}}}{n^{\frac{\sigma}{1-\sigma}} + [[\sum_{j=1}^{n} \alpha_j^{\frac{1-\sigma}{\phi+\sigma}}]^{\frac{\phi+\sigma}{1-\sigma}}]^{\frac{1}{1+\phi}}}$$

On the other hand, note that given (6) and a level of inequality aversion ε it is possible to write the EDEI as

$$[\sum_{i=1}^{n} {\alpha_i}^{1-\varepsilon}]^{\frac{1}{1-\varepsilon}} = n^{\frac{\varepsilon}{1-\varepsilon}}[1 - A_\varepsilon(\mathbf{y})]. \qquad (14)$$

Now, define $\widehat{\varepsilon} = 1 - \frac{1-\sigma}{\phi+\sigma}$. Given that, it is straightforward to rewrite the equilibrium winning probability as stated in the text of the Proposition. ■

The Proposition shows that it is possible to write relevant equilibrium variables as a function of the Atkinson index of inequality of the distribution of income within the group. For that, the inequality aversion parameter must be set as $\widehat{\varepsilon}$. This parameter $\widehat{\varepsilon}$ is increasing both in the degree of complementarity of efforts σ and the cost elasticity of effort ϕ and ranges between $-\infty$ and 2. Once we know the functional relationship between inequality and success in the conflict we can perform comparative statics on the equilibrium.

3.2 Comparative statics

Let us first analyze the question of how the level of inequality within the group affects its chances of prevailing over the external threat. To do so we use income transfers as a natural way of comparing distributions in terms of their inequality. We say that a transfer is *progressive* (*regressive*) when it is made from an individual i to an individual j such that $i > (<) j$ and such that the relative ranking of these two individuals remains unchanged after the transfer.

Proposition 3 *Given two distributions of income* $\mathbf{y}$ *and* $\mathbf{y}'$ *such that* $\mathbf{y}'$ *can be obtained after a sequence of progressive transfers, the group's winning probability* p^* *is lower under* $\mathbf{y}$ *than and* $\mathbf{y}'$ *if and only if* $\widehat{\varepsilon} \geq 0$.

The proof of this result is immediate from inspection of (13) and after using the fact that the Atkinson index of inequality is consistent with the Principle of transfers if and only if $\widehat{\varepsilon} \geq 0$.

This proposition shows that when members' efforts are substitutable enough or the cost of effort is linear enough, more unequal communities tend to fare better in conflict. On the other hand, when efforts are complementary enough and the cost of effort is convex enough, i.e. $\widehat{\varepsilon} \geq 0$, more egalitarian societies have a higher chance of prevailing over the external threat. The intuition for this result is easy to grasp: Complementarity of efforts discourages free-riding. All members must contribute if the group wants to have a fair chance of defeating threat. Poorer members are the ones who have less to gain from victory. Hence, the higher their income share, that is, the more egalitarian the group, the more they contribute. When efforts are substitutes, poorer members can free-ride in richer members who have more at stake in the conflict. So the higher the income share of the richer members the higher their efforts.

Regarding the cost, the more convex it becomes the less willing members are to contribute a substantial amount of effort. This deterrent effect is especially strong for richer members who shy away from making high contributions. More egalitarianism weakens this effect and helps the success of the group.

The properties of the Atkinson index allow us to perform some additional comparative statics

Proposition 4 *The group's equilibrium winning probability p^* decreases*

(i) When members of the group are replicated if and only if $\phi < 1$;

(ii) With the degree of complementarity of members' efforts σ.

Proof. For part (i) we use the Principle of Population of the Atkinson index. If the population is replicated the index remains invariant. We thus only need to check the term $n^{\frac{1-\phi}{1+\phi}}$ in the denominator of (13). It is clear that this term is increasing in n (and hence p^* is decreasing in n) if and only if $\phi < 1$.

For part (ii) note that the parameter σ only appears in $A_{\widehat{\varepsilon}}(\mathbf{y})$. Recall that p^* is decreasing in $A_{\widehat{\varepsilon}}(\mathbf{y})$. Given that for a given income distribution, the index is increasing in the inequality aversion parameter $\widehat{\varepsilon}$ and that $\widehat{\varepsilon}$ is increasing in σ, the winning probability p^* decreases as σ increases. ■

Esteban and Ray (2001) obtain that when the cost of conflict effort is convex enough, bigger groups are more effective in conflict, and the so-called "group size paradox" (Olson, 1965) is reversed. In our set up, increasing the size of the group is not a straightforward comparative statics exercise because the income distribution within the group changes as well. We are able to obtain an analogous result by employing the Principle of population and the Atkinson index: If the population is replicated, and so their income, the Atkinson index does not change, so any change in the winning probability is due to the direct change in n.

Part (*ii*) of the Proposition shows that when efforts are substitutes, that is, σ is relatively low, poorer members can free-ride on the efforts of richer members who have more at stake in the conflict. When efforts become more complements however, all members need to contribute in order to ensure that the group has a chance of prevailing against the threat. This incentivizes poorer members to contribute more. But these are precisely the members who have less to gain from winning the conflict. Complementarity thus reduces the effectiveness of the contributions by richer members because, as we can see from expression (8), it equalizes contributions across members. The success of the group thus rests on poorer members, whose effort is less intense than for richer members, making the group less effective.

Example 1: Tullock Contests Success Function: The Tullock CSF assumes that members' efforts are perfect substitutes, i.e. $\sigma = 0$. Two specifications have been mostly studied. The one with linear costs, i.e. $\phi = 0$ (Tullock, 1967; Baik, 1993) and the one with quadratic costs, i.e. $\phi = 1$ (Esteban and Ray, 1999).

The Tullock contest with linear costs corresponds to the case $\widehat{\varepsilon} \to -\infty$. In this case only the member with the highest valuation of victory would exert positive effort, member n in our case.

The group's equilibrium winning probability then boils down to

$$p^* = \frac{\alpha_n}{1 + \alpha_n},$$

so the income of agent n is the sole determinant of the success of the group. Note that in this case 1) the distribution of conflict efforts is rather unequal, i.e. $t_i = 0$ for all $i < n$; and 2) that poorer individuals might be potentially interested in transferring income to the richest individual in order to fuel her conflict effort.

The Tullock contest with quadratic costs corresponds to the case where $\widehat{\varepsilon} = 0$. In this case the Atkinson index of inequality is equal to zero for any distribution of income and the group's winning probability becomes just $p^* = 1/2$. Hence, the success of the group does not depend on how income is distributed within the group.

Example 2: The weakest-link technology (Hirshleifer, 1983). When members' efforts are perfect complements, i.e. $\sigma \to \infty$, only the minimum effort determines the success of the group. Hence

$$h(\mathbf{x}, n) = n \cdot \min\{x_1, ..., x_n\}.$$

This case corresponds to $\widehat{\varepsilon} \to 2$. In this case, all members exert the same level of effort in equilibrium. As we mentioned above, there exists a continuum of these equilibria . We will select the Pareto superior equilibrium, the one in which all members contribute $\overline{x}_1$. This would also be the selected equilibrium if one of the members were to move before the rest. The group's equilibrium winning probability in that case boils down to

$$p^* = \frac{n{\alpha_1}^{\frac{1}{1+\phi}}}{1 + n{\alpha_1}^{\frac{1}{1+\phi}}},$$

so the success of the group only depends on the share of income held by the worst-off member. This implies that 1) the distribution of conflict efforts is perfectly egalitarian; and 2) that richer individuals might be interested in transferring income to the worst-off member in order to boost her conflict effort.

4 Redistribution

4.1 Redistribution through conflict

In this section we study the relationship between income redistribution and conflict. We analyze two issues. First, how the presence of an external threat shapes the distribution of income within the group. Second, we explore the incentives of the group to engage in pre-conflict income redistribution.

As we saw in the previous section, starting from a peaceful situation where agents enjoy a share of the total income α_i the emergence of an external threat alters the distribution of that income. Members contribute in equilibrium a share $p^*(1-p^*)t_i$ of their income in order to defend their group and their incomes. The resulting distribution may be more or less egalitarian than the initial one. We next analyze whether this implicit taxation that conflict induces is *pro-rich* or *pro-poor*.

At this point it is necessary to define the Lorenz curve for the relevant distributions. The Lorenz curve of the initial income distribution L^{α} is just given by the income shares

$$L_j^{\alpha} = \sum_{i=1}^{j} \alpha_i \qquad j = 1, ..., n.$$

Similarly, the Lorenz curve for the cost distribution L^c is given by the shares

$$L_j^c = \sum_{i=1}^{j} \frac{{x_i}^{1+\phi}}{\sum_{i=1}^{n} {x_i}^{1+\phi}} = \sum_{i=1}^{j} \frac{\alpha_i^{2-\widehat{\varepsilon}}}{\sum_{i=1}^{n} {\alpha_i}^{2-\widehat{\varepsilon}}} \qquad j = 1, ..., n.$$

Finally, the Lorenz curve for the ex post income in case of victory $L^{\widehat{u}}$ is given by shares

$$L_j^{\widehat{u}} = \sum_{i=1}^{j} \frac{y_i(1-t_i)}{\sum_{i=1}^{n} y_i(1-t_i)} \qquad j = 1, ..., n.$$

By the Jakobson-Fellman Theorem (1976) it is possible to show that

$$L^{\widehat{u}} \geq L^{\alpha} \geq L^c \Leftrightarrow t_i \geq t_j \text{ for any } i > j.$$

This theorem together with inspection of expression (11) for t_i yield the following corollary.

Corollary 5 *The ex ante distribution of income Lorenz-dominates the ex post distribution of income if and only if* $\widehat{\varepsilon} \geq 1$

When efforts are complementary enough, the implicit redistribution caused by conflict is *pro-rich*. That is, the ex post income distribution is more unequal than the original distribution. This is because, when efforts are complementary, poorer members must contribute in order to secure a chance of success in the conflict. Therefore the profile of contributions is regressive. When efforts are substitutes, though, poorer members can free ride on the effort of richer members and the resulting profile of contributions is progressive: richer members contribute a higher share of their income that poorer members.

4.2 The effect of income redistribution

Let us now analyze the question of whether members of the group have incentives to engage in pre-conflict redistribution in order to improve its chances of prevailing over the threat. To this end we can exploit the properties of the Atkinson index.

Recall that the group winning probability p^* is decreasing in the Atkinson index of inequality. Because the properties of the index depend on the value of $\widehat{\varepsilon}$, transfers have different effects under different technologies.

Proposition 6 *The group's equilibrium winning probability p^* increases*

(i) *With a progressive transfer if $\widehat{\varepsilon} \geq 0$; The increase is larger the poorer the two members involved in the transfer.*

(ii) *With a regressive transfer if $\widehat{\varepsilon} < 0$. When $\widehat{\varepsilon} \in (-1, 0)$ this increase is larger the poorer the two members involved in the transfer. The opposite holds when $\widehat{\varepsilon} \leq -1$.*

Proof. The proof of this Proposition rests on the properties of the Atkinson index. When $\varepsilon \geq 0$,the index satisfies the Principle of transfers. In addition, the fact that $w'''(y_i) \geq 0$ implies that a given transfer between poorer agents is more effective in decreasing inequality than the same transfer made between richer individuals. This is called the Principle of diminishing transfers (Kolm, 1976).

Things are slightly less straightforward when $\widehat{\varepsilon} < 0$. In that case, $w'''(y_i) \geq 0$ if and only if $\widehat{\varepsilon} \leq -1$. Recall that in that case, the Atkinson index decreases only with regressive transfers. A non positive third derivative $w'''(y_i)$ implies here that such transfer is more effective in reducing the index when is made between two poorer individuals. To see this consider two individuals i and j such that $y_i = y_j + \Delta$ and an infinitesimal transfer d made from the latter to the former. The net increase in individuals utilities is just

$$\Delta w = [w'(y_i) - w'(y_j)]d = [w'(y_j + \Delta) - w'(y_j)]d.$$

For the impact on the index to be greater when the transfer is made between poorer it must be that Δw is decreasing y_j, which requires

$$w''(y_j + \Delta) < w''(y_j),$$

and thus that $w'''(y_i) < 0$. Finally, when the third derivative is non-negative the opposite property holds and transfers have a greater impact on the index if made between richer members. ■

The Proposition suggests that members may be interested in redistributing income voluntarily before the conflict takes place in order to increase their prospects of victory. When $\widehat{\varepsilon} > 0$, the group's winning probability increases with progressive transfers so richer members may be interested in transferring some of their income to poorer members in order to incentivise them to exert more effort. The opposite might happen when $\widehat{\varepsilon} < 0$ since in that case impact and cost technologies tend to depress poor agents' effort. They might be interested in transferring some income to richer and more active individuals within the group.

The following proposition characterizes whether these types of voluntary transfers will take place or not. We have to restrict ourselves to particular (but relevant) cases since the analysis of general cases is not analytically tractable.

Proposition 7 *In a Tullock contest with both linear or quadratic costs, i.e. $\widehat{\varepsilon} \to -\infty$ or $\widehat{\varepsilon} = 0$, no voluntary transfers take place. Under the weakest-link technology, i.e. $\widehat{\varepsilon} = 2$, there exists a income threshold $\overline{\alpha}$ such that any member with income share $\alpha_i > \overline{\alpha}$ is willing to make a transfer to the worst-off member of the group.*

Proof. Let us start with the cases in which voluntary transfers do not take place. When $\widehat{\varepsilon} \to -\infty$ the payoff of any member $i \neq n$ is

$$u_i = \frac{\alpha_n}{1+\alpha_n}\alpha_i Y \qquad i = 1, ..., n-1.$$

From here it is straightforward to check that no member has an incentive to transfer a share of its income to member n.

Similarly for $\widehat{\varepsilon} = 0$, the expected payoff of any member is

$$u_i^* = \frac{1}{2} y_i (1 - \frac{\alpha_i}{2(1+\phi)}),$$

which does not depend in the income shares of other members and is always increasing in α_i. Hence no member has any incentive to transfer part of its income to another member.

Finally, consider the case where $\widehat{\varepsilon} = 2$. The expected payoff for any member $i \neq 1$ is just

$$u_i = \frac{n\alpha_1^{\frac{1}{1+\phi}}}{1+n\alpha_1^{\frac{1}{1+\phi}}} Y(\alpha_i - \frac{1}{1+\phi}\frac{\alpha_1}{1+n\alpha_1^{\frac{1}{1+\phi}}}) \qquad i = 2, ..., n.$$

Knowing that $\alpha_i = 1 - \alpha_1 - \sum_{j \neq 1,i} \alpha_j$ then the impact of an infinitesimal transfer of income from i to member 1 on i's payoff is given by the derivative

$$\frac{\partial u_i}{\partial \alpha_1} = \frac{p^*(1-p^*)}{1+\phi}\frac{Y}{\alpha_1}(\alpha_i - \frac{1-p^*}{1+\phi}\alpha_1) - p^* Y (1 + \frac{1-p^*}{1+\phi}(1 - \frac{p^*}{1+\phi})).$$

And this derivative is positive if and only if

$$\alpha_i > \overline{\alpha} \equiv \alpha_1(1 + \frac{1+\phi}{1-p^*} + \frac{1-2p^*}{1+\phi}).$$

■

This Proposition shows that the incentives to redistribute income that an external confrontation creates have different strengths depending on the conflict and cost technologies. When $\widehat{\varepsilon} < 0$, a poorer member can increase the chance of victory of the group by transferring some of her income to a richer member, but her incentive to do so is rather small because she has a small stake in the confrontation. When $\widehat{\varepsilon} > 0$ progressive transfers increase the group winning probability, and richer members can find profitable to give away part of their income in order to enhance the conflict effort of poorer members. For the case $\widehat{\varepsilon} \to \infty$, sufficiently rich members are willing to transfer part of their income to poorest member in order to fuel her conflict effort. Note that such transfers are Pareto improving since all members benefit from them. Note also that as these transfers are made, the identity of the worst-off member changes and that changes the threshold $\overline{\alpha}$ as well.

5 Conclusion

The relation between inequality and social conflict has been subject to intense theoretical and empirical scrutiny in the last few years. Less attention however has been given to the relationship between external conflict and internal inequality. In this paper we have shown that the technology of conflict -protection or appropriation- plays a crucial role in that relation.

Egalitarian societies are more likely to prevail in a confrontation against an external group if the effort of their members are complementary. This technology encompasses cases such as military secrecy, defence of fortifications or modern armies where strength is very related to the lowest effort made by members of society. Given that technology, egalitarianism increases the chances of the group prevailing in the conflict because it increases the stakes of the poorer members. On the other hand, unequal societies are more effective in conflict when efforts of their members are substitutes. This scenario encompasses cases such as lobbying or the use of small mercenary armies. In this case, the community is more successful as richer members get richer because they have bigger stakes in the conflict.

We characterize the relationship between within-group inequality and conflict expenditures as a function of the Atkinson index of inequality. This allows us to express equilibrium variables as a relationship of inequality in a transparent manner and allows us to exploit the properties of the index when performing comparative statics. For a given distribution of income, more complementarity of efforts is bad for the community. Victory depends increasingly in the lowest effort, which is made by the poorest members of the group, which in turn are the ones with the smallest stakes in the conflict. Hence, more complementarity makes the success of the group rest on those individuals who have the lowest incentives to contribute. On the other hand, society can obtain an advantage in conflict as it becomes bigger provided that the cost of conflict is convex enough. This is in line with previous results (Esteban and Ray, 2001) showing that convexity of the cost is key for groups to overcome the collective action problem pointed out by Olson (1965).

More importantly, we show that members of society have incentives to engage in voluntary redistribution. This redistribution is aimed at increasing the incentives of other members to contribute more to the conflict effort. This incentives are asymmetric, however. We show that when more egalitarianism within the community makes it more likely to prevail, richer agents have incentives to transfer part of their income to poorer members. These transfers can be Pareto improving for the group. However, when more inequality makes the group more likely to prevail, poorer members do not have incentives to make transfers to richer members. This is because poorer members do not have much to gain from victory. Hence, we conclude that external threat constitutes a force that explains the progressive income redistribution that we observe in modern societies.

There are two limitations in our analysis that deserve further exploration. One is the assumption that the external threat is unitary. In that sense, conflict efforts could also be interpreted as efforts aimed at mitigating the effects of a natural catastrophe or at avoiding an epidemic. It would be interesting to explore the role of relative inequality between the threat and the group in the equilibrium. The incentives of a society to engage in income redistribution should also vary depending on the inequality of the out-group it is facing. The second limitation has to do with the completely decentralized nature of interactions within the group. Members contribute efforts voluntarily and, if willing to, transfer part of their income to other members. In reality, states have traditionally requested these efforts from its subjects, often using coercion. Taxation has arisen as an institutional mechanism aimed at redistributing income in order to help societies to wage war. We intend to explore more centralized mechanisms in our future research.

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