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War Finance and the Modern State

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Abstract

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1 Introduction

At times of war, having ready access to money is essential. Failing to pay or furnish troops, to improve technology or to lobby to form alliances may quickly bring to defeat. Tilly (1992), Besley and Persson (2009) and Besley and Persson (2010) argue that the success at war of a state depends crucially on its capacity to raise funds through taxation. Implicitly, in their argument they assume that the state does not to have access to financial markets. From a theoretical view point, if a state has access to financial markets, their conclusion may not hold, i.e., taxation may induce defeat in warfare. If victory

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gives control of valuable resources, a state without any wealth could borrow all necessary funds to wage war, and then acquire these resources. This state would honor its debt only in case of victory, by sharing the booty of war with the creditors. It seems interesting then to explore how the details of the relationship between a state and its creditors can affect the intensity of warfare, and the chances of victory.

European states have been at war most of their time. Due to the discontinuity and unpredictability of war spending, states have never been able to finance wars with their own savings: they had to rely heavily on loans (Kindleberger (1993)). Then, it seems reasonable to think that the nature of the modern state has been shaped by the necessity to effectively fund warfare. More specifically, it seems reasonable to assume that institutions which allowed states to borrow efficiently at times of war are likely to have endured, and to have been imitated by other states. According to the definition in Ferguson (2001), the modern state is characterized by the square of power: taxation, parliament, central banking and public debt. These four institutions are the key ingredients of modern public finance. This work explores the effectiveness and complementarity of these institutions in enhancing the capacity of a state to raise funds at times of war. Through an argument of positive selection, this work suggests that these institutions may be widespread today because of having bred success at warfare in earlier modern times.

1.1 Modeling the finance of war

War. War is modeled as a contest: two states compete by spending resources to increase their probability of acquiring the control of a prize, which is fixed in value. This is a popular workhorse model for representing conflict, which besides of its obvious limitations captures both the high riskiness of war and its inefficiencies. The financial market is modeled as a continuum of investors and analyzed under very different circumstances: forced loans, free entry and perfect collusion. Contracts among the states and their creditors are assumed to be perfectly enforced as far as the relevant variables are observable. While this representation of the financial market is clearly simplistic, it seems reasonable to postpone the analysis of more realistic and complicated cases as imperfect enforceability of contracts to future studies.

Institutions. Taxation is modeled as increasing the wealth of the state, hence its liability towards its creditors. The parliament is seen as an institution which monitors public spending on behalf of the creditors, allowing the state
to credibly commit funds to warfare\textsuperscript{1}. The central bank is modeled as the state making public issuances of debt, announcing to all creditors the total amount of borrowed funds\textsuperscript{2}. Public debt is seen as the state borrowing in relatively efficient financial markets by issuing tradeable instruments whose price reflects its likelihood of default.

1.2 Theoretical results

Existence and uniqueness of equilibria. Contests are usually studied under the assumption of continuous expenditures. In our model, if the expenditures are continuous, the equilibrium is always unique and interior, i.e., both states borrow funds, spend in warfare, and have some chance of victory. In the paper, we consider also the case of binary expenditures, showing that in this case there is the possibility of multiple equilibria. A state locked in a bad equilibrium can get out of it by developing taxation, or parliament and central bank. This result is in contrast with Berner
gard and Vesperoni (2013), which shows that, in contests with binary expenditures, but without financial markets, the equilibrium is always unique.

Taxation. Taxation increases the chances of victory only if the financial market is efficient, in the sense of interest rates being elastic to default risks. On the contrary, if interest rates are inelastic, taxation decreases the chances of victory. This result is in contrast with Yates (2011) on the role of limited liability in contests, and in contrast with Besley and Persson (2009) and Besley and Persson (2010) on the role of taxation in warfare.

Parliament and central bank. Parliament and central bank are individually ineffective for funding warfare. Only if both are simultaneously in place, central bank and parliament are effective, becoming a perfect substitute for taxation. This result is in contrast with Wärneryd (2000) on the role of moral

\textsuperscript{1}This is also the main assumption behind the thesis of North and Weingast (1989) on the Glorious Revolution, where the empowerment of the British parliament prevented the strategic default of the King, solving a problem of credit rationing.

\textsuperscript{2}Historically, the creation of central banks coincided with the rationalization and standardization of public debt. Central banks would be the favorite intermediary between government and financial market, managing the efficient issuance of public debt. In principle, governments do not need central banks to issue public debt, since they could directly deal with their creditors. In this work we argue that the efficient issuance of public debt requires credible announcements of the total amount of borrowed funds. While the government lacks the incentive to truthfully report this amount, a central bank controlled by the main creditors of the government instead would have such credibility.
hazard in contests. It is also in contrast with the literature on the Glorious Revolution which developed from North and Weingast (1989), and in particular with Cox (2011), arguing that without the creation of the Bank of England the empowerment of the British parliament would have been ineffective in funding warfare.

**Monopolist investor.** A monopolist investor (or a perfectly collusive financial market) always lends to both states. This is somehow counter intuitive, since backing both sides the investor incurs the certain loss of the default of the loser. Lending to both sides is convenient because, if the opponent is highly armed, a state is willing to pay higher interest for its loan. A monopolist investor always lends less than a competitive financial market would do. Moreover, under certain conditions, the monopolist investor wants the states to adopt some of the institutions of modern public finance.

### 1.3 History of the modern state

The theoretical results listed above provide an explanation for some historical events related to the development of the modern state. Let us quickly see them.

**Taxation did not develop in the middle ages because limited liability gave a strategic advantage in warfare.** If interest rates are inelastic to default risks a state without taxation has a lower expected cost of borrowing, since it defaults on the debt in case of defeat. Having a lower expected cost, the state without taxation would spend more aggressively in warfare. The development of financial markets in early modern times made interest rates more elastic to probabilities of default: borrowing became more and more costly for states with weak taxation. Developing taxation became convenient around the 17th century, since it allowed to borrow more cheaply in the rising European financial markets. This is a possible explanation of the rise of taxation in modern times, alternative to the thesis of the *military revolution* by Roberts (1954) which argues that technological changes in warfare induced higher costs and hence higher taxation.

**Parliaments and central banks developed simultaneously because they complemented each other in manipulating the financial market.** Without central bank the state borrows small loans separately from each investor. With central bank, the state makes public issuances of debt, announcing to all investors the total amount of borrowed funds. The more a state spends in warfare the lower its probability of default, and hence the lower the
interest rate. In order for the announcement to affect the interest rate the
state has also to commit to spend all borrowed funds in warfare, but this may
not be the case due to a problem of moral hazard. In early modern times
only wealthy individuals were represented in parliament, the same individuals
who were among the main creditors of the state. Empowering the parliament
meant giving them monitoring powers towards the management of their in-
vestments. With parliament in place the state could then credibly commit
to spend all borrowed funds in warfare and exploit its market power in the
financial market.

The Baring Family funded both sides of the Napoleonic Wars to
charge higher interest rates. At the beginning of the 19th century the
House of Baring was virtually a monopolist in financing the British and the
French governments. During the Napoleonic Wars the Barings financed war
efforts of both sides: they brokered the issuance of most British debt and lent
to United States the necessary funds to buy Louisiana from France (Ziegler
(1988)). The Barings may have lent to both sides to induce an arm race and
hence inducing France and England to accept higher interest rates.

The welfare state developed only in the 20th century because parlia-
ments and central banks were already in place. Before the development
of parliaments and central banks it was essential for the states to devote most
taxation to honor war debts, in order to decrease the risk premium and bor-
row cheaply. The combination of parliament and central bank revealed to be
as effective as taxation in funding warfare, relaxing the budget constraint of
the states. States had various reasons to increase their expenditure in welfare,
but they could afford that only once parliaments and central banks were in
place.

The paper develops as follows. Section 2 briefly explains the historical back-
ground, focusing on the development of financial markets and comparing the
institutional changes in England and France. Section 3 exposes the related
literature in economics and in other social sciences. Section 4 explains the
model. Sections 5, 6 and 7 respectively expose the results concerning three
different states of the financial market: forced loans, free entry and perfect
collusion. Section 8 concludes, discussing historical events with the help of
theoretical results.
2 Historical background

This section is a very brief summary of the historical development of European financial markets and of the institutions of the modern state. Each paragraph of this section summarizes information taken from the sources cited at its end.

Financial markets started developing in the late middle ages at the merchants’ fairs. Afterwards wealthy merchants such as the Medici from Florence specialized in international payments. Their bills of exchange started being traded in commercial centers, and the process led to relatively developed financial markets in the 17th century (De Roover (1999)).

At that time Amsterdam had established itself as the most vibrant trading center and financial capital was abundant there. Because of this several European states raised their funds for warfare there. During the 18th century London progressively became the financial capital of the world, leaving Amsterdam behind. The final blow to the supremacy of Amsterdam came with the Napoleonic Wars, when capitals fled to London during the French occupation of the Netherlands (Cassis and Collier (2006)).

The financial opportunities and risks of the intense warfare of the 18th century caused an increasing concentration of financial capital in the hands of few merchant bankers located in London. In the early 19th century the issuance of public debt was virtually monopolized by the Baring and the Rothschild families. Towards the end of the century their supremacy was challenged by the rise of universal banks, which financed industry and warfare with the deposits of small investors, and the increasing ability of states to directly issue their debt on financial markets without the brokerage of merchant banks (Ziegler (1988) and Ferguson (1999)).

Modern taxation, parliament, central bank and public debt have been developed by European states in different situations, from the 17th century onwards. The rest of the section describes and compares the institutional developments of England and France, since these states have been among the first in modernizing their institutions and other states followed their example.

In 17th century England, during the Stuart monarchy, the Crown would borrow funds through short term debt. This short term debt was functionally equivalent to taxation, since the lenders were often forced into accepting the terms dictated by the Crown. As an example, in 1626 King Charles I borrowed around 250,000 Pounds threatening of imprisonment whoever would not accept
its terms. Besides of coercing creditors into forced loans, the Crown did not systematically default on these loans. Defaults were frequent, but mostly due to insolvency. While the creditors could be forced into accepting the terms of the Crown in lending, they would strongly react to systematic defaults (Ashton (1960), Kenyon (1986) and Brewer (1988)).

The combination of taxation, parliament, central bank and public debt as square of power appeared for the first time in 17th century England. During the Thirty Years War the Stuarts increased taxes considerably and improved the efficiency of their collection by creating a bureaucracy devoted to it: the Excise. The Glorious Revolution of 1688 transferred effective monitoring and veto powers concerning the public budget to the parliament. By 1715 the parliament would guarantee that a specific tax would be assigned to service each new loan to the Crown. The Bank of England was established in 1694. The English public debt became increasingly long term and tradeable. The first government bond was issued in 1693 and contributed to finance the Nine Years war. The first Consol was issued in 1751 and quickly became the most popular instrument for British war financing (Brewer (1988), North and Weingast (1989), Ferguson (2001), Wells and Wills (2000), Sussman and Yafeh (2006) and Cox (2011)).

Like England, France felt the financial pressure of the Thirty Years War, but reacted differently. Instead of developing a bureaucracy for tax collection and issuing tradeable public debt on the financial market, France raised funds by selling the rights to collect present and future taxes on a territorial basis to so called Tax Farmers. While this contract resembles issuance of public debt, the price of it is unlikely to be close to a competitive one. This system made tax collection to be extremely inefficient, to the point that in the 18th century while France had roughly two times the GDP of England it collected less than half of its tax revenue. Moreover the fragmentation and arbitrariness of the rights assigned to Tax farmers made it difficult to represent their interests in parliament, which would have put some discipline on the management of public funds (White (1989), Weir (1989), Quinn (2001) and Ferguson (2001)).

From the Glorious revolution to the beginning of French revolution England and France participated in a series of conflicts which always saw them at the opposite sides: the Nine Years War (1688-1697), the Spanish Succession War (1701-1714), the Austrian Succession War (1741-1748), the Seven Years War (1756-1763) and the American War for Independence (1776-1783). In most of these wars England clearly dominated France, confirming itself as the strongest power in the international political arena. While from 1688 to 1788 England
never defaulted, France repeatedly failed to honor its debts\textsuperscript{3}. The financial market perceived the higher reliability of England as a borrower compared to France and reacted accordingly: while the interest rate on English public debt sharply declined from 14 per cent in 1693 to 3 per cent in 1726 France would be systematically charged higher interest rates, paying a risk premium due to its higher likelihood of default (Weir (1989), Velde and Weir (1992), White (1995), Sargent and Velde (1995), Ferguson (2001) and Sussman and Yafeh (2006)).

The institutions of England and France converged somewhat during the 19th century, but never completely. After the failed attempt of John Law with the Caisse d’Escompte in 1776, it was Napoleon who founded the Banque de France just before starting one of the most ambitious wars of Europe. Besides this, Napoleon did not rely much on bond issuances to fund its wars. This happened only much later, with the Crimean and Franco-Prussian wars. During the 19th century England progressively introduced direct taxation of income, in order to lower the indirect taxes which were distorting international trade. While the Tax Farmers’ system relied mostly on direct taxation, after the Revolution France reorganized its tax system choosing to rely mostly on indirect taxation (Ferguson (2001) and Dincecco (2009)).

3 Related literature

This paper argues that the institutional setting of the state has been shaped by the necessity of being efficient in warfare. This thesis is not new to social scientists. Max Weber’s analysis of the rise of bureaucracy devoted much attention to the spillovers between civilian and military hierarchy, arguing that the need of developing an efficient army is the strongest factor in the evolution of bureaucracy and the development of state capacity. Otto Hintze’s work on the rise of the Prussian state developed a structural model of the evolution of state power under the pressure of external military threat. Roberts (1954)’s thesis of the military revolution argues that major technological changes in warfare in the 16th century forced the states to adapt their institutional setting to new needs. Tilly (1992)’s work on the rise of fiscal capacity argues that

\textsuperscript{3}The defaults of France took the form of reimbursement suspension or cuts in interest payments. The first default concerns the debts from the Spanish Succession War: after directly defaulting on part of the debt in 1715 and indirectly defaulting by devaluing the currency in 1723, with the interest rate cut of 1726 France defaulted on 70 percent of its war debt. The second and third defaults concern the Seven Years War: in 1759 and 1770 France suspended reimbursements and converted floating debt into perpetuals.
states introduced taxes under the pressing need of funding war. This thesis has been expanded and formalized in economics in Besley and Persson (2009) and Besley and Persson (2010).

More specifically this work focuses on the financial aspect of warfare, an aspect which has been often neglected in the works mentioned above. The works of Tilly (1992), Besley and Persson (2009) and Besley and Persson (2010) on fiscal capacity and warfare are challenged by it. If states have access to the financial market it is not clear why they should necessarily increase taxes to fund wars, instead of issuing public debt. As a counterexample to their thesis in the next sections it is shown that when interest rates are inelastic increasing taxation lowers the chances of victory. On the contrary if interest rates are sufficiently elastic taxation increases the chances of victory. Since the historical development of financial markets is likely to have increased the elasticity of interest rates to default risks, this result suggests that states developed taxation in the 17th century because of the contemporary development of finance. This explanation is alternative to Roberts (1954)’s thesis of the military revolution, arguing that the rise of the modern state is not due to technological changes in warfare but due to the development of financial markets.

Economic historians have been working on the financial aspect of warfare and its relation with the institutions of the state. North and Weingast (1989)’s seminal work on the Glorious Revolution argues that the empowerment of the parliament allowed England to commit honoring its public debt, solving a problem of credit rationing due to strategic default. Among the numerous following contributions on the topic, Cox (2011) argues that, instead of solving a problem of credit rationing, the Glorious Revolution gave monitoring powers to the parliament, solving a moral hazard problem over the management of public funds. This work follows the interpretation of Cox (2011), but claims that the empowerment of the parliament would not have been effective in financing warfare without the creation of the Bank of England few years later.

From a theoretical point of view this study builds on previous contributions in contest theory (see Konrad (2009) for a review). More specifically this work builds on previous contributions on the roles of liability and moral hazard in

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4The development of financial markets may have increased the elasticity of interest rates to default risks for several reasons. Firstly, finance became more and more disentangled from politics, and hence non monetary compensations for risk taking became monetized in the interest rate. Secondly, the development of financial markets is likely to have increased the quality of information regarding the degree of solvency of the debtors, which makes interest rates more elastic to risks of default.
contests. While contests with full liability have been widely studied before, there are only few works on the role of limited liability (see Skaperdas and Gan (1995), Wärneryd (2000) and Yates (2011)). The role of moral hazard is studied in Wärneryd (2000). None of these works studies the issue of funding expenditures in the financial market. The only other work in contest theory which focuses on the financial aspects of conflict is Slantchev (2012) which analyses how leverage may prevent contestants from settling instead of fighting. This work shows that introducing financial issues in contests may reverse the results. Yates (2011) argues that limited liability induces higher expenditures. With financial market this may not be the case, since limited liability induces default in case of defeat and hence a risk premium on the interest rate. Wärneryd (2000) argues that moral hazard induces lower expenditures. This is not the case with financial market, since with or without moral hazard the investors must be compensated for the risks they take. Bernergård and Vesperoni (2013) argues that in discrete contests the equilibrium is always unique, while with financial markets there is the possibility of a state being locked in a bad equilibrium.

This setting is close to the literature on corporate finance and the product market. In this literature the financial choices of firms affect their revenues on the product market, and these revenues have feedback effects on their financial choices (see Kovenock and Phillips (1995), Kovenock and Phillips (1997), Showalter (1999) and Schroth and Szalay (2010)). Similar feedback effects are at play in this work. Besides of this similarity there are stark differences between competing in the product market and competing in a contest, and these differences radically change the analysis. The good contended by firms in the product market is aggregate demand, which is not fixed in value as the prize of a contest. Moreover the product market is not a winner-take-all situation as a contest, since firms always get some share of the market.

4 The model

Two states, players 1 and 2, fight over a prize of common value $q > 0$. Each state $i \in \{1, 2\}$ borrows resources $y_i \geq 0$ in the financial market to fund its war expenditures $x_i \geq 0$. For each state $i \in \{1, 2\}$, the cost of borrowing is $R_i y_i$, where $R_i$ is the gross interest rate on its debt. The more the states spend the higher their probability of winning $P^i(x_i, x_{-i})$. The win probabilities$^5$ take

$^5$Skaperdas (1996) axiomatized this form of win probabilities, showing that it essentially follows from independence from irrelevant alternatives in changing the number of contest-
the form

\[ P^i(x_i, x_{-i}) = \frac{x_i + \beta_i}{x_1 + \beta_1 + x_2 + \beta_2} \]  

(4.1)

if \( x_1 + \beta_1 + x_2 + \beta_2 > 0 \) and \( 1/2 \) if \( x_1 + \beta_1 + x_2 + \beta_2 = 0 \), where \( \beta_i \geq 0 \).

The parameter \( \beta_i \) is a stock of war effort, which state \( i \) has accumulated in a pre-play period, such as a standing army.

The financial market is constituted by a continuum of investors, represented by player 3. Their aggregate wealth \( W_3 \geq 2q \) has to be allocated in three financial instruments: a riskless asset \( z_3 \) and the debts of the two states \( z_1 \) and \( z_2 \), where \( z_i \geq 0 \) and \( z_1 + z_2 + z_3 = W_3 \). Their gross interest rates are \( R_i \geq 1 \) and the one of the riskless asset is normalized to 1. Three cases will be analyzed: forced loans, free entry and perfect collusion. In case of forced loans the states always obtain the demanded funds \( z_i = y_i \) at gross interests \( R_i = R_3 \). In case of free entry in the financial market the investors always supply the states with the demanded funds \( z_i = y_i \) and the gross interests \( R_i \) are identified by a no profit condition. In case of perfect collusion the investors act as a monopolist, setting quantities \( z_i \) and the gross interests \( R_i \).

Contracts between states and their creditors are perfectly enforced in the financial market as far as the relevant variables are observable. State \( i \) borrows \( y_i \) and spends \( x_i \leq y_i \) in warfare. By contract state \( i \) is expected to spend all borrowed funds in warfare, but if it does not have parliament then its expenditure \( x_i \) is not observed by the creditors and hence not enforced. In these circumstances the state could divert funds to different activities whose gross return is equal to the one of the riskless asset\(^6\). By developing parliament the state gives monitoring powers to the creditors, committing itself to spend all the borrowed funds in warfare.

Let us see how taxation, parliament and central bank enter in the model. The states own illiquid wealth \( W_i \geq 0 \) which they use as collateral for their debts. Their wealth is either 0 or \( \bar{W} \geq q \). States are normally without any wealth, \( W_i = 0 \). Developing taxation they build illiquid wealth by acquiring the stream of future fiscal revenues, \( W_i = \bar{W} \).

\(^6\)If the return of these different activities was larger than \( R_3 \) the state would be in a situation of arbitrage. If it was lower than \( R_3 \) moral hazard would increase the expenditures in warfare, making the results of this work even starker.
A state without central bank acts in two periods. In the first period it borrows \( y_i \) resources in the financial market and in the second period it spends \( x_i \) resources in warfare. As mentioned before, a state with parliament commits itself to \( x_i = y_i \). For a state without parliament instead the two choices remain separated. A state with central bank has an additional pre-play period, when it credibly announces to the financial market the total amount of funds \( y_i \) it intends to borrow.

All players are risk neutral\(^7\). Their payoffs are monetary and cannot be negative. If a contingency brings a player to a negative payoff, then the player is insolvent and defaults on the part of debt that cannot be repaid. The payoff of the representative investor is

\[
\pi_3(z) = \sum_{i=1}^{3} z_i R_i N D_i
\]

where \( N D_i \) is an indicator function which takes value 0 in case of default of asset \( i \) and 1 otherwise. Since by definition asset 3 never defaults then \( N D_3 \) is always 1. If state \( i \) has taxation then \( N D_i \) is also always 1, and hence its expected gross interest rate is \( E[R_i] = R_i \). If state \( i \) does not have taxation then \( N D_i \) is 1 if and only if state \( i \) wins the war, and hence its expected gross interest rate is \( E[R_i] = P_i(x_i, x_{-i}) R_i \). As a tie breaking rule, if the payoff of the investor is the same from investing in asset \( i \in \{1, 2\} \) and asset 3, the investor strictly prefers to fulfill the demands of funds, \( z_i = y_i \).

The payoff of state \( i \) is

\[
\pi_i(x, y) = W_i + y_i - x_i - R_i y_i + q V_i
\]

if \( W_i + y_i - x_i - R_i y_i + q V_i \geq 0 \) and 0 otherwise, where \( V_i \) is an indicator function which takes value 1 in case of victory of state \( i \) and 0 otherwise. As a tie breaking rule, if the payoff of state \( i \) is the same from borrowing \( y_i \geq x_i \) state \( i \) strictly prefers to borrow \( y_i = x_i \).

Since a state would never\(^8\) borrow at costs \( R_i y_i \) higher than the prize \( q \), by

\(^7\)The section on perfect collusion studies also the case of the representative investor being risk averse. The results in the sections on forced loans and free entry do not hold if the states are risk averse, except of the positive effect of the central bank when the state has parliament and does not have taxation.

\(^8\)This is true only if the state aims to win, which is generally the case. This is not the case only under forced loans and only if the state does not have parliament and does not
equation 4.3 if \( W_i = 0 \) state \( i \) honors its debt in case of victory and defaults otherwise, while if \( W_i = \bar{W} \) it always honors its debt.

Let us see how taxation and parliament affect the expected payoffs of the states. If state \( i \) has taxation then it always honors its debt. Because of this by contract it has to spend the borrowed funds in warfare, \( x_i = y_i \), with or without parliament. Its expected payoff is

\[
E[\pi_i(x, y)] = \bar{W} - R_i y_i + P^i(y_i, x_{-i})q
\]  

(4.4)

which is equivalent to the expected payoff of an all pay contest with cost \( R_i y_i \) and prize \( q \) (see Konrad (2009)).

Consider state \( i \) without taxation. If it has parliament then it is committed to \( x_i = y_i \) and in case of defeat it defaults on its debt due to insolvency. Its expected payoff is

\[
E[\pi_i(x, y)] = P^i(y_i, x_{-i}) (q - R_i y_i)
\]  

(4.5)

which is equivalent to the expected payoff of a player in a winner pay contest with cost \( R_i y_i \) and prize \( q \) (see Yates (2011)).

If a state without taxation does not have the parliament then it has the possibility of diverting borrowed funds in alternative activities with gross return 1. In case of defeat it always defaults due to the expenditure \( x_i \) not being verifiable. Its expected payoff is

\[
E[\pi_i(x, y)] = y_i - x_i + P^i(x_i, x_{-i}) (q - R_i y_i)
\]  

(4.6)

which is equivalent to the expected payoff of a player in an all pay contest with cost \( x_i \) and prize \( (q - R_i y_i) \).

The next three sections analyze the model under different states of the financial market: forced loans, free entry and perfect collusion. In the section on free entry the model is analyzed also under the assumption of binary efforts, showing how this induces multiple equilibria. In the sections on forced loans and free entry the analysis is focused on levels of expenditure of the states, not on their expected payoffs.

have taxation. As shown in section 4, in this setting the state does not want to win but to exploit arbitrage. Besides of this, even in this case the conclusions regarding default and liability hold.
In these sections two institutional settings are said to be equivalent if they induce the same level of expenditures. The analysis focuses on war expenditures because the expected payoffs are difficult to compare. Because of this the results do not give any information regarding the desirability of institutions for the states. Even if it cannot be argued that states consciously developed institutions, the results can still explain their success. Suppose states adopt institutions at random. If a certain institutional setting induces higher expenditures then the state adopting it has higher chances of victory and hence it is more likely that such institutional setting endures.

5 Results: forced loans

If the financial market operates through forced loans then the states set the amounts of their loans $y_i$ and their gross interest rates are always 1. More precisely given certain demands of funds $y_1$ and $y_2$ a forced loans market equilibrium is defined as for all $i \in \{1, 2\}$ the supply of funds $z_i$ and the gross interest rate $R_i$ being such that $z_i = y_i$ and $R_i = 1$. In this setting the gross interest rates are inelastic to probabilities of default. Loans are forced because if a state defaults with some probability then no investor would voluntarily be its creditor, since it would strictly prefer to invest in the risk free asset.

Given the financial market is in forced loans market equilibrium, the model is solved by subgame perfect Nash equilibrium. The proof of the following Proposition is in appendix A. The case of states not having taxation and not having parliament is not further analyzed.

Proposition 5.1 Assume forced loans. A state without taxation and without parliament is in a situation of arbitrage. If states have taxation or have parliament there always exists a unique internal equilibrium, and if $\beta_i = 0$ there are no corner solutions.

Having central bank is irrelevant in this setting. central bank does not directly affect the payoffs of the players, but manipulates the equilibrium interest rates by affecting their expectations. Since the interest rates are fixed to 1 then the central bank is irrelevant. A state with taxation is always fully liable towards

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9This kind of arbitrage is unlikely in the historical period considered. This is because the investors would react by not lending to the state, as shown in Drehichman and Voth (2011) regarding the creditors of Philip the II. While the investors may be forced in accepting interest rates which do not perfectly compensate for default risks, they are unlikely to accept to lend to a state which systematically diverts the borrowed funds into other activities and strategically defaults.
its creditors, hence by contract has to spend all borrowed funds in warfare. It follows that with taxation having parliament is irrelevant.

**Proposition 5.2** Assume forced loans. *Having central bank is always irrelevant. For a state with taxation having parliament is irrelevant.*

Consider state $i$ with parliament. If state $i$ has taxation its expected payoff is as in equation 4.4, where $R_i = 1$ due to forced loans. By the concavity of the expected payoff its best reply $BR_i(x_{-i})$ is identified by the first order condition

$$P_1(x_i, x_{-i})q = 1$$

If state $i$ does not have taxation and has parliament, its expected payoff is as in equation 4.5, where $R_i = 1$ due to forced loans. By the concavity of the expected payoff its best reply $BR_i(x_{-i})$ is identified by the first order condition

$$P_1(x_i, x_{-i})q = P_1(x_i, x_{-i})x_i + P(x_i, x_{-i})$$

The LHSs of equations 5.1 and 5.2 are the same and decreasing in $x_i$, due to the concavity of the win probability. If taxation induces higher expenditures, then the RHS of equation 5.1 must be lower than the one of equation 5.2, which is

$$1 < P_1(x_2, x_1)x_2 + P(x_2, x_1)$$

By the functional form of the win probabilities this implies $x_{-i} + \beta_1 + \beta_2 < 0$, which is never the case. By Proposition 5.2 having central bank is always irrelevant, hence the same holds with central banks.

**Proposition 5.3** Assume forced loans. *If a state has parliament, taxation always induces lower expenditures.*

The results from Propositions 5.2 and 5.3 are combined and represented in figure 1, and summarized in Theorem 5.1.

**Theorem 5.1** Under forced loans, a state without taxation and without parliament is in a situation of arbitrage. For every other institutional configuration, there exists a unique equilibrium, which is interior. In equilibrium, a state is more likely to win without taxation than with taxation.
Figure 1: Effects of institutions on war spending under forced loans ($\emptyset = \text{no institutions}, P = \text{parliament}, B = \text{central bank}, T = \text{taxation}$). The cases $\emptyset$ and $B$ are not represented because they induce arbitrage. The relative magnitude of the expenditures $x_i$ under different institutional settings is purely ordinal.

6 Results: free entry

If there is free entry in the financial market then the gross interest rates are identified by a no profit condition and the states set the amounts of their loans $y_i$. More precisely given certain demands of funds $y_1$ and $y_2$ a free entry market equilibrium is defined as for all $i \in \{1, 2\}$ the supply of funds $z_i$ and the gross interest rate $R_i$ being such that $z_i = y_i$ and $E[R_i] = 1$, where the expectation operator is conditional on the information of the representative investor in the first period.

Given the financial market is in free entry market equilibrium the model is solved by subgame perfect Nash equilibrium. The proof of the following Proposition is in appendix $B$.

**Proposition 6.1** Assume free entry. There always exists a unique internal equilibrium. For $\beta_i < q/4$ there are no corner solutions.

If a state has taxation then it never defaults, and hence by the no profit condition it always pays gross interest 1 on its debt. Having parliament is irrelevant in this case, since being fully liable towards the investors the state spends all borrowed funds in warfare. Having central bank is also irrelevant, since by the no profit condition the interest rate is equal to the risk free one and hence constant.

**Proposition 6.2** Assume free entry. For a state with taxation having parliament or central bank is irrelevant.
Let us consider states without taxation and see how parliaments and central banks affect their expected payoffs. If states without taxation do not have central banks the model has two periods. In the first period the states collect \( y_i \) funds in the financial market. Each investor lends to the states a small fraction of these funds and is not informed of their total amount. Each investor believes the states to play their equilibrium strategy in the second period and the no profit condition to hold, and hence lends at the equilibrium interest rates. The states take such interest rates as given and play their strategies accordingly.

If a state without taxation has central bank then a pre-play period is added to the model, in which the state credibly communicates to the investors its commitment to raise \( y_i \) funds. If both states have central banks then their communication is simultaneously done in the pre-play period. If a state does not have parliament then it cannot credibly commit to spend in warfare anything different than its subgame perfect expenditure of the last period. Because of this for a state without parliament having central bank is irrelevant. If the state which makes the announcement has parliament, it credibly communicates its commitment to spend \( y_i \) in warfare. This information affects the expectations of the investors regarding the probabilities of default and hence the no profit condition \( E[R_i] = 1 \). Making the announcement the state takes into account how it affects the equilibrium interest rates and plays its strategy accordingly: the state is a price maker in the financial market. By the no profit condition the state expects the gross interest rate on its debt to be the inverse of its win probability \( R_i = [P^i(y_i, x_{-i})]^{-1} \). Taking this into account its expected payoff in equation 4.5 becomes

\[
E[\pi_i(x, y)] = P^i(y_i, x_{-i}) \left( q - \frac{y_i}{P^i(y_i, x_{-i})} \right)
\]

which is equivalent to the expected payoff of a state with taxation in equation 4.4.

**Proposition 6.3** Assume free entry. For state without parliament having central bank is irrelevant. Given the financial market is in equilibrium, having central bank and parliament is equivalent to having taxation.

Let us see how, for a state without central bank and without taxation, having parliament affects its expenditures. Suppose state \( i \) is without taxation and without parliament, having a problem of moral hazard. Its expected payoff is as in equation 4.6. Since the interest rate increases with the probability of default then, in contrast with the case of forced loans, state \( i \) is not in a
position of arbitrage. It follows that in equilibrium states have no incentive to borrow more than they spend in warfare, hence in the first period $y_i = x_i$ is optimal. The first order condition of state $i$ is

$$P^i_1(x_i, x_{-i})(q - R_i y_i) = 1 \quad (6.1)$$

and from the no profit condition of the investors the equilibrium interest rate is $R_i = [P^i(x_i, x_{-i})]^{-1}$.

Suppose now state $i$ is without taxation but with parliament. Its expected payoff is as in equation 4.5. Its first order condition is

$$P^i_1(y_i, x_{-i})(q - R_i y_i) = P^i(y_i, x_{-i}) R_i \quad (6.2)$$

and from the no profit condition of the investors the equilibrium interest rate is $R_i = [P^i(y_i, x_{-i})]^{-1}$. At the equilibrium interest rate the RHS of equation 6.2 is the same as the RHS of equation 6.1, equal to 1. Moreover given $y_i = x_i$ the LHSs are the same. It follows then that at the equilibrium interest rates the first order conditions in equations 6.1 and 6.2 are equivalent. The rest follows from Proposition 6.2.

**Proposition 6.4** Assume free entry. Given the financial market is in equilibrium, for a state without central bank having parliament is irrelevant.

Let us now see the effect of having taxation on the expenditures of a state with parliament and without central bank. Suppose state $i$ has taxation. Its best reply is identified by

$$P^i_1(x_i, x_{-i})q = 1 \quad (6.3)$$

Suppose instead state $i$ does not have taxation, then its best reply is identified by equation 6.2. The LHSs of equations 6.3 and 6.2 are equal and by the concavity of the win probabilities they are decreasing in $x_i$. It follows that in equilibrium if a state spends more without taxation then the RHS of equation 6.3 must be higher than the one of equation 6.2, which means

$$1 > R_i [P_1(x_2, x_1)x_2 + P(x_2, x_1)]$$

Recall that by the no profit condition the equilibrium interest rate of state $i$ is $R_i = P(x_i, x_{-i})^{-1}$. After few manipulations this leads to
Figure 2: Effects of institutions on war spending under free entry (\(\emptyset = \) no institutions, \(P =\) parliament, \(B =\) central bank, \(T =\) taxation). The relative magnitude of the expenditures \(x_i\) under different institutional settings is purely ordinal.

\[
1 > 1 + \frac{P_1(x_2,x_1)x_2}{P(x_2,x_1)}
\]

which is never the case. It follows that having taxation induces higher expenditures. The rest follows from Propositions 6.3 and 6.4.

**Proposition 6.5** Assume free entry. If a state does not have central bank or does not have parliament having taxation induces higher expenditures.

The results from Propositions 6.2, 6.3, 6.4 and 6.5 are combined and represented in figure 2, and summarized in Theorem 6.1.

**Theorem 6.1** Under free entry, for every institutional configuration, there exists a unique equilibrium, which is interior. In equilibrium, a state is more likely to win with taxation than without taxation. In equilibrium, parliament and central bank are individually ineffective in enhancing victory, but if together in place are as effective as taxation.

Let us consider a binary effort space \(x_i \in \{l, h\}\), where \(h > l > 0\). For simplicity assume \(\beta_i = 0\). Suppose states do not have taxation and do not have central bank, maximizing their expected payoffs taking the interest rate as given. As discussed before, in this case having parliament is irrelevant. Assume the financial market to be in equilibrium. Regarding symmetric equilibria, \(x_1 = x_2 = h\) is an equilibrium if and only if

\[
\frac{1}{2}(q - 2h) \geq \frac{l}{l + h} [q - 2l]
\]
where, given the opponent plays $h$, the LHS is the expected payoff from playing $h$ and the RHS the expected payoff from playing $l$.

On the other hand $x_1 = x_2 = l$ is an equilibrium if and only if

$$\frac{1}{2}(q - 2l) \geq \frac{h}{l + h} [q - 2h]$$

Regarding asymmetric equilibria, $x_i = h$ and $x_{-i} = l$ is an equilibrium if and only if

$$\frac{h}{h + l}(q - h - l) \geq \frac{1}{2} \left[q - (h + l) \frac{l}{h}\right]$$

$$\frac{l}{h + l}(q - h - l) \geq \frac{1}{2} \left[q - (h + l) \frac{h}{l}\right]$$

both hold. Consider $q = 1$, $h = 1/4$ and $l = 1/16$. With these parameter values there are three equilibria in pure strategies: the symmetric one in which both states exert high effort and the two asymmetric ones where one state exerts high and the other low effort. The reason for the multiplicity of equilibria is that the interest rates adjust with the efforts of the states: if one exerts high (low) effort then the interest rate is low (high), making it convenient to actually exert such high (low) effort.

Let us compare these results with the situation in which the states have parliaments and central banks, or have taxation. It is easy to verify that the game is dominance solvable and it always has a unique symmetric equilibrium. The equilibrium is $x_1 = x_2 = l$ if

$$\frac{1}{2}(q - 2l) > \frac{h}{h + l}(q - h - l)$$

and $x_1 = x_2 = h$ otherwise\(^\text{10}\). Consider again the parameter values $q = 1$, $h = 1/4$ and $l = 1/16$. In this case the unique equilibrium is $x_1 = x_2 = h$.

Consider the asymmetric situation in which state 1 has taxation, or has parliament and central bank, and state 2 does not have taxation, and does not have parliament or central bank. For state 1 the game is dominance solvable. For instance if $q = 1$, $h = 1/4$ and $l = 1/16$ the optimal effort is always $h$.

\(^{10}\)If the LHS is equal to the RHS then every strategy profile is an equilibrium.
For state 2 the self fulfilling mechanism is at play, and may generate multiple equilibria. For instance if $q = 1$, $h = 1/4$ and $l = 1/16$ there are two equilibria: one in which state 2 plays $h$ and one in which plays $l$.

This simple example with binary efforts shows how one state not having taxation and not having parliament or central bank generates multiplicity and asymmetry of equilibria, while both states having taxation or parliament and central banking induces uniqueness and symmetry of equilibria. More specifically a state without taxation and without central bank or without parliament may fall in a bad equilibrium which is a self fulling trap: it borrows little because the interest rate is high, and the interest is high because it borrows little. By developing taxation or central bank and parliament the state eliminates the bad equilibrium.

**Theorem 6.2** Assume free entry and binary efforts. If both states have taxation or parliament and central bank there is a unique equilibrium which is symmetric. The lack of taxation and the lack of parliament or central bank in at least one state induces multiple equilibria.

### 7 Results: perfect collusion

Under perfect collusion the representative investor acts as a monopolist, choosing the quantities $z_i$ and interest rates $R_i$ which maximize its expected payoff in equation 4.2 subject to some constraints. If state $i$ has parliament or has taxation then the constraint of the investor regarding state $i$ is its participation constraint, where the outside option of state $i$ is participating to the conflict without any funding. If state $i$ does not have parliament and does not have taxation then the constraints of the investor regarding state $i$ are its participation constraint and its incentive compatibility constraint, where state $i$ should be prevented from diverting funds in alternative activities.

Let us start from the case of state $i$ having parliament. Since the expected payoff of the investor is increasing in the interest rate $R_i$ of state $i$ its participation constraint is binding in equilibrium. The participation constraint of state $i$ with taxation is

$$P_i(z_i, z_{-i})q - R_i z_i \geq P_i(0, z_{-i})q$$

and the interest rate $R_i$ which makes it binding is

21
The participation constraint of state $i$ without taxation is

$$P_i(z_i, z_{-i}) = q R_i z_i$$

(7.2)

and the interest rate $R_i$ which makes it binding is

$$R_i^T(z_i, z_{-i}) = \frac{q}{z_i}(P_i(z_i, z_{-i}) - P_i(0, z_{-i}))$$

(7.4)

Consider the expected revenue $E[R_i] z_i$ of the investor from state $i$ when its participation constraint is binding. It is easy to verify from equations 7.2 and 7.4 that the one with taxation is equal to the one without taxation. It follows that the investor is indifferent about the states having taxation.

Suppose the investor is risk averse. While a state with taxation always repays its debt, a state without taxation repays its debt only in case of victory. We have just seen that lending to a state without taxation gives the same expected revenue than lending to a state with taxation, due to the risk premium paid by the state without taxation. Since the expected revenues are the same but the revenue from a state without taxation is uncertain, by Jensen inequality a risk averse investor is better off lending to a state with taxation. It follows that a risk averse investor wants the states to have taxation.

**Proposition 7.1** Assume perfect collusion and states having parliaments. A risk neutral investor is indifferent about taxation. A risk averse investor wants the states having taxation.

Let us solve for the equilibrium investments $z_i$, assuming for simplicity that the investor is risk neutral. We have already seen that the expected revenue of the investor is the same for states with taxation or without taxation. By equations 7.2 and 7.4 the expected payoff of the investor when the participation constraints are binding is $E[\pi_3(z_1, z_2)] =

$$q - \sum_{i=1}^{2} P_i(0, z_{-i})q + W_3 - z_1 - z_2$$

(7.5)

which is concave in both investments $z_i$. By first order conditions the optimal investments are
\[ z_i^* = \sqrt{q\beta_i - \beta_1 - \beta_2} \] (7.6)

hence the investor lends to both states, more to the one with the lowest \( \beta_i \). It is easy to verify that the state with the smallest standing army \( \beta_i \) is also the most likely to win in equilibrium. The following Proposition summarizes the results.

**Proposition 7.2** Assume perfect collusion and states having parliaments. A risk neutral investor lends to both states. The investor lends more to the state with the lowest \( \beta_i \) which is also the most likely to win.

From the previous section it is easy to verify that when there is free entry in the financial market the state with the largest \( \beta_i \) is better off, hence states have an incentive to develop standing armies. Interestingly the opposite happens when the financial sector is monopolized: states are better off not having a standing army.

Let us compare the equilibrium expenditures with the ones in case of free entry in the financial market. By Proposition 6.5 if there is free entry aggregate expenditures are maximized if both states have taxation, or parliament and central bank. It is easy to verify that the best replies are

\[
BR_i(x_i) = \sqrt{(x_{-i} + \beta_{-i})q - x_{-i} - \beta_1 - \beta_2}
\]

Suppose states have taxation, or have parliament and central bank. With perfect collusion by equation 7.6 the equilibrium investment \( z_i \) is equal to the best reply \( BR_i(0) \) with free entry. It follows that, given this institutional setting, the equilibrium expenditures are always higher with free entry than with perfect collusion. The following Proposition summarizes the results.

**Proposition 7.3** Assume perfect collusion and states having taxation, or having parliament and central bank. Their expenditures are always higher with free entry than with perfect collusion.

Let us consider the case with states not having parliaments and not having taxation. The investor has the additional problem of providing the incentives not to divert funds into alternative activities. The incentive compatibility constraint of state \( i \) without taxation is

\[
P_i(z_i, z_{-i})(q - R_i z_i) \geq P_i(0, z_{-i})(q - R_i z_i) + z_i \] (7.7)
The incentive compatibility constraint is less demanding than the participation constraint if and only if in equilibrium the RHS of equation 7.7 is smaller than the one of equation 7.3, which requires

\[ P_i(0, z_{-i}) q \geq P_i(0, z_{-i})(q - R_i z_i) + z_i \]

If the incentive compatibility constraint is less demanding then in equilibrium the participation constraint should be binding. If the participation constraint is binding then the interest rate is as in equation 7.4. It follows that the incentive compatibility constraint is less demanding than the participation constraint if

\[ P_i(0, z_{-i}) q \geq \frac{P_i(0, z_{-i})^2 q}{P_i(z_{1}, z_{-i})} + z_i \]  

(7.8)

Let us assume that the participation constraint is binding, solve for the optimal investments and then verify if condition 7.8 holds. If the participation constraint is binding the problem is equivalent to the one with parliaments. We have seen before that in equilibrium the loan to state \( i \) is \( z_i = \sqrt{q\beta_i} - \beta_1 - \beta_2 \). Given these investments levels, condition 7.8 becomes

\[ \sqrt{q\beta_i} - \beta_i \left[ 1 + \frac{\sqrt{q\beta_i} - \beta_i}{\sqrt{q\beta_{-i}} - \beta_{-i}} \right] \geq \sqrt{q\beta_{-i}} - \beta_1 - \beta_2 \]  

(7.9)

It is easy to verify that if \( \beta_1 = \beta_2 \) the LHS of equation 7.9 is equal to the RHS, hence the scheme is an equilibrium and both the participation and the incentive compatibility constraint are simultaneously binding. Suppose instead \( \beta_2 > \beta_1 \). After few manipulations of the condition for state 1 we get to

\[ \sqrt{q\beta_2} - \sqrt{q\beta_1} - \beta_2 < 0 \]

It is easy to verify that, if \( \beta_2 < q \), there always exist \( \beta \) such that for all \( \beta_1 < \beta \) the condition does not hold. This means that for \( \beta_1 \neq \beta_2 \) the scheme that is optimal with parliaments is not necessarily optimal without parliaments, since the state with the smallest standing army \( \beta_i \) may lack the incentive to spend the borrowed funds in warfare. Notice that the scheme that is optimal with parliaments maximizes the expected revenue of the investor, since the participation constraints are binding. It follows that, for \( \beta_1 < \beta \), the investor wants state 1 to have parliament.
Proposition 7.4 Assume perfect collusion and states not having parliaments. If \( \beta_1 = \beta_2 \) the investor is indifferent about parliament. If \( \beta_1 \neq \beta_2 \) and \( \beta_2 < q \) there exists \( \hat{\beta} \) such that for all \( \beta_1 < \hat{\beta} \) the investor wants state 1 to have parliament.

8 Discussion

This work studies how warfare shapes the institutional setting of the states, focusing on taxation, parliament and central banking. The higher the war expenditures of a state the higher its chances of victory, and hence the higher the likelihood of its institutional setting persisting in time and being imitated by other states. The issue is analyzed in an environment where states fund their expenditures by borrowing in a financial market. Three types of financial market are studied: forced loans, free entry and perfect collusion. Let us briefly summarize the results, first focusing on their contributions to contest theory and then on their historical relevance.

8.1 Contest theory

The literature on contests implicitly assumes that players fund their expenditures with their own wealth. This work introduces the possibility of borrowing funds in the financial market. Perhaps the most important result is that, in a contest where expenditures are funded in the financial market, the equilibrium is always unique and interior. Moreover, the analysis shows also that some established results in contest theory are reversed when contestants finance their efforts through borrowing.

While Yates (2011) shows that limited liability induces higher expenditures in contests, this work shows that the opposite is true when contestants borrow in a financial market with free entry. When states do not have taxation they are limitedly liable towards their creditors, defaulting in case of defeat. By the no profit condition their interest rates are higher than the risk free one, to compensate for the default risk. By developing taxation they become fully liable and hence they borrow more cheaply, ending up spending more and hence increasing their chances of victory.

While Wärneryd (2000) shows that moral hazard decreases expenditures in contests, this work shows that this is not the case when contestants borrow their funds in a financial market with free entry. Moral hazard is relevant only
if a state does not have taxation, since it defaults in case of defeat. With or without parliament, by the no profit condition the expected cost of borrowing is equal to the risk free interest rate. Since without parliament the opportunity cost of spending is also equal to the risk free interest rate, then parliament does not affect expenditures in equilibrium.

This work shows that, with a binary expenditure space, if players borrow their funds in a financial market with free entry there are multiple equilibria. This result is opposite to Bernergård and Vesperoni (2013), which shows that without financial markets the equilibrium is unique. Since by the no profit condition the interest rate is high (low) when expenditures are low (high), the state may get locked in the self fulfilling trap of borrowing little at high interest. This result relies on states not having taxation, otherwise the interest rate is the risk free one and hence constant. Moreover it relies on the state not having central bank or not having parliament, otherwise the state is price maker in the financial market and anticipates that the interest rate decreases by borrowing highly. It follows that a state in a bad equilibrium would jump to a better equilibrium by developing taxation or central bank and parliament.

8.2 History of the modern state

Besides of the contributions to contest theory, the results of this work give insight on some events in the history of war finance and the development of the modern state. European states have constantly been at war since the middle ages. Because of the high magnitude and unpredictability of war spending, states could not finance wars through their savings and had to rely heavily on loans. From the 17th century onwards states started developing common institutions which would drastically influence their ability to borrow funds for warfare: taxation, parliament and central bank.

8.2.1 Taxation

States started developing systematic taxation only in the 17th century. At that time financial markets became sufficiently efficient, such that interest rates would reflect default risks. This work shows that when interest rates are inelastic to risks of default states without taxation have higher chances of victory. On the other hand if interest rates are elastic then taxation becomes advantageous. The intuition is that taxation has two opposite effects on expected costs: they may increase because the state always pays back, and they may
decrease because pays less by avoiding the risk premium. If interest rates are inelastic to risks of default the first effect dominates, while if they are elastic the opposite holds. These results suggest that in the middle ages, before the development of financial markets, states would not have systematic taxation because it would decrease their chances of victory. In the 17th century, when financial markets reached sufficient maturity, taxation became advantageous and states with it would be the most successful in warfare.

The idea that modern states developed systematic taxation because of the rise of financial markets enters in several debates. The literature on fiscal capacity (see Tilly (1992), Besley and Persson (2009) and Besley and Persson (2010)) claims that states developed taxation to finance wars, but it does not take into account the possibility that states could borrow the necessary funds. This literature assumes implicitly that states are credit rationed, having to fund most of current war expenditures through current taxes. Besides the fact that states always borrowed funds to finance warfare\textsuperscript{11}, states were surely more credit constrained in the middle ages than in modern times, hence following the theory of fiscal capacity it is not clear why they did not develop systematic taxation already in the middle ages. The theory of the military revolution (Roberts (1954)) provides a possible answer, arguing that the magnitude of war spending increased considerably around the 17th century, due to technological innovations in warfare. This explanation is far from obvious, since it is not clear why technological innovations in warfare should necessarily induce higher war spending. If a state can induce the same damage to its enemy at a lower cost and there are no returns from increasing the inflicted damage, then its war spending should actually decrease.

8.2.2 Parliament and central bank

States started developing parliaments and central banks in the 18th and 19th centuries, and tended to develop the two institutions at the same time. This work shows that parliament and central bank are individually ineffective. Only if both in place they give an advantage in warfare. Parliament alone is ineffective, because as argued before moral hazard does not affect equilibrium expenditures in this case. Central bank alone is ineffective, because its announce-

\textsuperscript{11}In fact it could be argued that the superior ability to borrow funds for funding warfare is at the core of the nature of the state. In principle all men are approximately equal in their coercitive power. One of them may dominate only if manages to hire others to dominate others. This leader should be the one who is the most credible in effectuating these payments once in power.
ments to the financial market are not credible unless the parliament enforces them. On the other hand the two institutions together allow the state to manipulate the interest rates, exploiting the fact that borrowing high amounts lowers the risk premium.

This thesis enters in the debate on the effects of the Glorious Revolution on war financing developed from the seminal work of North and Weingast (1989). North and Weingast (1989) assume that England was credit rationed before 1688, due to the lack of commitment of the Crown to not expropriate its creditors by systematically defaulting. This assumption has been criticized providing evidence of the fact that at that time the property rights of the lenders were enforced to some degree. For instance Cox (2011) assumes that, as in this work, contracts are perfectly enforced in the financial market as far as the relevant variables are observable. By modeling war as a lottery with exogenous probability of victory, Cox (2011) shows that England may have had an incentive to develop ministerial responsibility through the Glorious Revolution. This work builds on the framework in Cox (2011) but models war as a contest, taking into account how the probability of victory may depend on war spending of both sides of the conflict. Taking this into account, this work shows how the monitoring powers of the parliament are ineffective unless a central bank is in place. In line with this result, the Bank of England was created only few years after the Glorious Revolution, possibly in order to exploit the complementarity of parliament and central bank in funding the incoming war with France.

States started developing the welfare state in the early 20th century, when central banks and parliaments were effectively in place. This work shows that when central bank and parliament are both in place they are a perfect substitute for taxation in warfare. The intuition is that when a state has central bank and parliament it can manipulate interest rates. When credibly announcing its war expenditures to the financial market it expects by the no profit condition that its interest rate decreases with expenditures. Because of this its equilibrium expenditures are equal to the ones of a state with taxation. This result suggests that once central banks and parliaments were in place states did not need anymore to devote taxation to guarantee the repayment of loans, and hence could divert funds to welfare services without inducing any disadvantage in warfare.
8.2.3 Collusive financial market

When the Baring family virtually monopolized the financial markets during the Napoleonic Wars, it lent funds both to France and England which were at war with each other. This work shows that a monopolist investor always lends to both sides of the conflict. This result is somehow counterintuitive, since when states do not have taxation one of them defaults with certainty. Besides of this certain loss the monopolist funds both sides because it lowers their reservation utility: the more the opponent is armed the higher the need of a state of getting armed. By lending to both sides the monopolist maximizes its expected revenue, charging higher interest rates.

The analysis of the model with the monopolist investor suggests also an alternative explanation for the development of taxation and parliament in modern times. Under some mild conditions it is in the interest of the investor to lobby for them to be in place. While a risk neutral investor is indifferent about taxation, if risk averse it would always want taxation in place. The intuition is that the reservation utility of the states does not depend on their degree of liability, since it is based on participating in the conflict with zero expenditures. It follows that the expected revenue of the investor does not depend on taxation, although the revenue from a state with taxation is certain while without taxation it is uncertain. If the investor is risk averse it always prefers the certain revenue from taxation. Regarding parliament, while with symmetric standing armies the investor is always indifferent about parliaments, if the standing armies are sufficiently asymmetric the investor wants the weaker state to have parliament. The intuition is that, while the weaker state would have the incentive to borrow, it would lack the incentive to spend the funds into warfare. This is because the weaker state is the one receiving the most funds, but also the one having the lowest outside option.

9 Appendix A

Consider a state without parliament and without taxation. Suppose the state spends no funds in warfare, $x_i = 0$. The expected cost of borrowing $y_i$ is $P_i(0, x_{-i})y_i$ which is always smaller than the return $y_i$ from diverting these funds in the alternative activities. It follows that the state is in a situation of arbitrage, since the expected cost is always lower than the return.

The rest of the appendix deals with existence and uniqueness of the equilibrium under forced loans when states have taxation or parliament. Consider both
states having taxation. Under forced loans their expected payoffs in equation 4.4 are equivalent to an all pay contest with cost $x_i$ and prize $q$. The existence and uniqueness of the interior equilibrium follows from Szidarovszky and Okuguchi (1997).

Consider both states not having taxation and having parliament. Under forced loans their expected payoffs in equation 4.5 are equivalent to a winner pay contest with cost $x_i$. The existence and uniqueness of the interior equilibrium follows from Yates (2011).

Consider state 1 having taxation and state 2 not having taxation and having parliament. The payoff of state 1 is the one of an all pay contest with cost $x_i$ and the one of state 2 of a winner pay contest with cost $x_i$. Let us consider corner solutions. For $\beta_i = 0$ the expected return of state $i$ from increasing its expenditure at $x_i = 0$ is always larger than the cost, hence there are no equilibria with $x_i = 0$. Let us consider interior solutions. Both best replies are such that $\lim_{x_i \to 0} BR_i(x_i) = 0$, which means that the best reply to $x_{-i} = 0$ is arbitrarily small. The best reply of player 1 is hill shaped\textsuperscript{12} and concave. The best reply of player 2 is always increasing and concave. Suppose the best replies do not cross. Given they both start at 0 and one is being always increasing and the other hill shaped this is impossible. It follows that there always exists an equilibrium which is an interior solution. Suppose the best replies cross in two points. This requires the best reply of one of the two states being convex in some interval, which is a contradiction. It follows that there is a unique interior equilibrium.

## 10 Appendix B

This appendix deals with existence and uniqueness of the equilibrium under free entry. Let us start considering internal solutions. Consider both states having taxation, or having parliament and central bank. By Proposition 6.3 their expected payoffs are the same and equal to equation 4.4, hence equivalent to an all pay contest with cost $x_i$ and prize $q$. The existence and uniqueness of the interior equilibrium follows from Szidarovszky and Okuguchi (1997).

Consider both states not having taxation and not having central bank. They may have parliament or not. By Proposition 6.4 if the financial market is in equilibrium their best replies are the same and implicitly defined by equation

\textsuperscript{12}More precisely, its first derivative is null in a unique point $x_{+i}$ which is the maximizer of the best reply.
6.1. By the functional form of the win probabilities this requires \( x_1 + \beta_1 = x_2 + \beta_2 \), and hence that the equilibrium interest rates are equal to 2. It follows from Szidarovszky and Okuguchi (1997) that the best replies cross in a unique point and hence the interior equilibrium exists and is unique.

Consider state 1 having taxation and state 2 not having taxation and not having central bank. The first order condition of state 1 is

\[
\frac{x_2 + \beta_2}{(x_1 + \beta_1 + x_2 + \beta_2)^2} \cdot q = 1
\]

which gives the best reply function \( BR_1(x_2) = \sqrt{(x_2 + \beta_2)q} - x_2 - \beta_1 - \beta_2 \). This best reply is hill shaped and its maximum value is \( q/4 - \beta_1 \). It is easy to verify that \( BR_1(x_2) \) is hill shaped\(^{13}\) and concave. Assuming the financial market in equilibrium, the best reply of state 2 is implicitly defined by the first order condition

\[
\frac{x_1 + \beta_1}{(x_1 + \beta_1 + x_2 + \beta_2)^2} \cdot \left( q - \frac{x_1 + \beta_1 + x_2 + \beta_2}{x_2 + \beta_2} \cdot x_2 \right) = 1
\]

By the best reply of state 1 after few manipulations the first order condition of state 2 becomes

\[
\left( \sqrt{q} - \sqrt{x_2 + \beta_2} \right) \left( \sqrt{q} - \frac{x_2}{\sqrt{x_2 + \beta_2}} \right) = \sqrt{(x_2 + \beta_2)q}
\]

This equation has always at least two positive roots of \( x_2 + \beta_2 \), one smaller and the others larger than \( q \). All roots are candidates for equilibria, but they need to satisfy other conditions. By the first order conditions of the two states it is easy to verify that in equilibrium \( x_1 + \beta_1 > x_2 + \beta_2 \). Since the maximum value of \( BR_1(x_2) \) is \( q/4 - \beta_1 \) it follows that in equilibrium \( x_1 + \beta_1 \) is always smaller than \( q \). Since \( x_1 + \beta_1 > x_2 + \beta_2 \) then \( x_2 + \beta_2 \) is necessarily smaller than \( q \). We can conclude that there exists a unique interior equilibrium.

Let us consider corner solutions. A necessary condition for a corner solution is that one state has null expenditures. Suppose state 1 has taxation or has parliament and central bank. If state 2 has null expenditures then the best reply of state 1 is \( BR_1(0) = \sqrt{\beta_2q} - \beta_1 - \beta_2 \). Suppose state 2 has taxation or central bank and parliament. State 2 has null expenditures if and only if

\(^{13}\)More precisely, its first derivative is null in a unique point \( x^*_2 \), which is the maximizer of the best reply.
\[
\frac{x_1 + \beta_1}{x_1 + \beta_1 + x_2 + \beta_2} q < 1
\]

Given state 1 plays \( BR_1(0) \) the condition of state 2 is fulfilled if and only if \( \beta_2 > q/4 \). Suppose state 2 does not have taxation and does not have central bank or parliament. As for the previous case, state 2 has null expenditures if and only if

\[
\frac{x_1 + \beta_1}{x_1 + \beta_1 + x_2 + \beta_2} q < 1
\]

Given state 1 plays \( BR_1(0) \) the condition of state 2 is fulfilled if and only if \( \beta_2 > q/4 \).

Suppose now that state 1 does not have taxation and does not have parliament or central bank. Since under free entry state 1 has a higher marginal cost than with taxation, it follows that \( BR_1(0) \) is smaller than with taxation. Because of this the condition on \( \beta_2 \) for the existence of corner solutions is weaker than before. It follows that for \( \beta_2 < q/4 \) there are no corner solutions.

References


